Specifications for a Linear Network Simultaneously Noise and Available-Power Matched

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***Abstract*— This letter addresses the problem of designing a linear lossy input matching network for low-noise amplifiers so that the source impedance can deliver its available power and correspond to the minimum noise figure of the driven stages. The differences between lossless and lossy networks are highlighted because matching circuits are usually considered to be lossless when designing an amplifier. After stating the assumptions, a solution to the problem of the minimum number of elements fulfilling the requirements is developed. The result explains why the standard distributed approach often fails to cope with minimum noise specifications when practical elements are considered.**

I. Introduction

The most desirable input matching circuit for a microwave active device should allow the source to deliver all its available power and simultaneously be the impedance corresponding to the minimum noise figure of the cascaded stages (Fig. 1). This letter addresses this issue and presents some theoretical results about the design of a real lossy input matching circuit. It is noticeable that the device $F_{\min}$ can still be achieved if a series feedback is applied to the transistor: the lossy input matching stage will increase the overall $F_{\min}^{ov}$, but a proper choice of the series feedback element can decrease the device $F_{\min}$ [1], [2] so that $F_{\min}^{ov} \simeq F_{\min}$. The chained stages are described by

$$\boldsymbol{C}^{IA} = \begin{bmatrix} R_n^{IA} & \widetilde{\rho_n^{IA*}} \\ \widetilde{\rho_n^{IA}} & g_n^{IA} \end{bmatrix} = \boldsymbol{C}^I + \boldsymbol{T}^I \boldsymbol{C}^A \boldsymbol{T}^{I+} \tag{1}$$

$$\boldsymbol{T}^{IA} = \begin{bmatrix} A^{IA} & B^{IA} \\ C^{IA} & D^{IA} \end{bmatrix} = \boldsymbol{T}^I \boldsymbol{T}^A. \tag{2}$$

$\boldsymbol{C}$'s are correlation matrices [3], $\widetilde{\rho_n^{IA}} = \rho_n^{IA}\sqrt{R_n^{IA} g_n^{IA}}$ where ρ_n^{IA} is the correlation coefficient of the stage, $\boldsymbol{T}$'s are transmission matrices, and the superscripts refer to the input matching circuit (I), to the following active network (A), and to the cascade of the two (IA). * and $^+$ are, respectively, the conjugate and the Hermitian conjugate operation. Thus, noise parameters change nonlinearly as functions of the input stage (1), while the signal matrix $\boldsymbol{T}^{IA}$ is linearly dependent on the input matching network (2), once the active stage $\boldsymbol{T}^A$ is defined. Further stages are neglected in (1) because they follow the active device [4].

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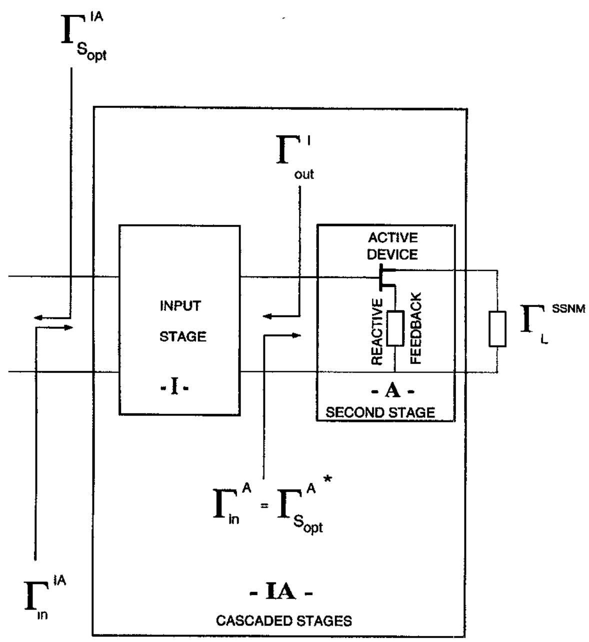

Fig. 1. Input-matching stage cascaded with a microwave amplifier and the assumptions for the design.

II. Assumptions

Consider Fig. 1. The following assumptions are made.

1) The stages are linear.
2) The source impedance is Z_o and the scattering parameters are normalized to Z_o.
3) The amplifier is assumed to be simultaneously signal and noise matched, i.e., $\text{SSNM}^A = \Gamma_{\text{in}}^A - \Gamma_{S_{\text{opt}}}^{A^*} = 0$.

SSNM defines a measure of how close the power match is to the condition for minimum noise figure. The condition $\text{SSNM}^A = 0$ is achievable in microwave low-noise amplifiers by a proper choice of the load $\Gamma_L = \Gamma_L^{\text{SSNM}}$ [5], usually after making use of a feedback element [1].

A proper choice of the load is necessary in order to get a SSNM condition at the input port of the device; this technique usually fails if applied to a device without feedback. If this condition is not achieved, the input matching network cannot simultaneously provide two different values—i.e., $\Gamma_{\text{in}}^{A^*}$ and $\Gamma_{S_{\text{opt}}}^A$—at the design frequency. The SSNM condition of the amplifier allows for the matching circuit to deliver its available power and a simultaneous noise and input match to be transferred to the input port of the cascaded stages, i.e., $\text{SSNM}^{IA} = 0$. The underlying assumption is that the overall design is carried out in two steps, the design of the amplifier and the design of the input matching circuit. If the amplifier's matrices $\boldsymbol{C}^A$ and $\boldsymbol{T}^A$ are known, the analysis can be focused on the input matching stage and simplifies the design problem. Different results might be expected if both the matching network and the amplifier had to be designed at the same time, e.g., by letting the feedback element vary. A full analytical approach as the following Sections, however, describe seems too complicated due to the inherent nonlinear nature of (1).

III. Requirements

At the design frequency, the input matching circuit must satisfy the following requirements:

$$\Gamma^{IA}_{S_{\text{opt}}} = 0 \tag{3a}$$

$$\Gamma^{IA}_{\text{in}} = 0 \tag{3b}$$

$$\Gamma^{I}_{\text{out}} = \Gamma^{A}_{S_{\text{opt}}}. \tag{3c}$$

The system of equations (3) implies:

a) the source impedance to correspond to the optimum source reflection coefficient of the driven stages, so that the minimum noise figure of the cascaded stages is achieved;
b) the source to deliver all its available power to the driven stages;
c) the output port of the matching circuit to deliver all its available power to the next active stage.

Notice that (3a) implies a noise requirement that is usually neglected at the early stage of the design. Equations (3b) and (3c) are equivalent in the case of lossless reciprocal networks, but not for practical lossy networks.

IV. Discussion

As described in Appendix A, system (3) can be restated as

$$\begin{aligned} 0 = {} & Z_o(g^I_n + R^A_n|C^I|^2 + 2\Re e[\widetilde{\rho^A_n} C^{I^*} D^I] + g^A_n|D^I|^2) \\ & - Y_o(R^I_n + R^A_n|A^I|^2 + 2\Re e[\widetilde{\rho^A_n} A^{I^*} B^I] + g^A_n|B^I|^2) \end{aligned} \tag{4a}$$

$$\begin{aligned} 0 = {} & \Im m[\widetilde{\rho^I_n} + R^A_n A^{I^*} C^I \\ & + \widetilde{\rho^A_n}^* B^{I^*} C^I + \widetilde{\rho^A_n} A^{I^*} D^I + g^A_n B^{I^*} D^I] \end{aligned} \tag{4b}$$

$$\begin{aligned} 0 = {} & [1 + (\Gamma^A_{S_{\text{opt}}})^*]A^I + [1 - (\Gamma^A_{S_{\text{opt}}})^*](B^I Y_o) \\ & - [1 + (\Gamma^A_{S_{\text{opt}}})^*](C^I Z_o) - [1 - (\Gamma^A_{S_{\text{opt}}})^*]D^I \end{aligned} \tag{4c}$$

$$\begin{aligned} 0 = {} & [1 + \Gamma^A_{S_{\text{opt}}}]A^I - [1 - \Gamma^A_{S_{\text{opt}}}](B^I Y_o) \\ & + [1 + \Gamma^A_{S_{\text{opt}}}](C^I Z_o) - [1 - \Gamma^A_{S_{\text{opt}}}]D^I. \end{aligned} \tag{4d}$$

The system (4) is a compact set of nonlinear equations at a fixed frequency; there are seven unknown input stage parameters: four of them refer to its transmission matrix, A^I, B^I, C^I, and D^I; three refer to its noise behavior, R^I_n, g^I_n, and $\widetilde{\rho^I_n}$. These seven unknowns are not independent.

1) If the input matching circuit is passive, then a plain expression between noise and signal parameters is obtainable as Appendix B demonstrates.
2) Suppose the input stage is an ordinary distributed matching circuit—a transmission line and a stub. Once the substrate has been chosen, the length and width of the transmission line and of the stub are the only independent variables. These four variables set up both the signal (the transmission $\boldsymbol{T}^I$ matrix) and the noise (the correlation $\boldsymbol{C}^I$ matrix) performance of the stage [6].
3) Assume the input stage is made of lumped RLC components: then, it is possible to work out the signal and noise parameters of the input stage as functions of these components.

There is no assumption about the passive or active, distributed or lumped nature of the input stage in writing (3). The relation between the noise and signal parameters of the input stage, however, has to be known, so that the expansion (4) may be restated as a function of the unknown circuit elements.

The input stage noise parameters may be expressed as functions of the complex unknowns A^I, B^I, C^I, and D^I. Three complex equations form system (3): there are more unknowns than equations. If the circuit has to be reciprocal, however, the determinant of $\boldsymbol{T}^I$ must be one. A reciprocal matching circuit must provide four degrees of freedom for its $\boldsymbol{T}^I$ matrix; each is responsible for a complex matrix term. A lumped circuit must contain resistors in order to get complex elements in $\boldsymbol{T}^I$. Therefore, either a simple stub plus transmission-line matching circuit or a lossless network cannot fulfill the goals (3).

For instance, if a single lossy transmission line is considered as input matching circuit, the only unknown is its length l. Its correlation matrix is [6]

$$\boldsymbol{C}^I = \begin{bmatrix} \frac{1}{2} Z_o \sinh(2\alpha l) & \sinh^2(\alpha l) \\ \sinh^2(\alpha l) & \frac{1}{2} Y_o \sinh(2\alpha l) \end{bmatrix}$$

where α is the attenuation in Np/m. Its transmission matrix satisfies $|\boldsymbol{T}^I| = \cosh^2(\gamma l) - \sinh^2(\gamma l) = 1$ where $\gamma = \alpha + j\beta$ and β is the phase constant in rad/m. After substituting the transmission line parameters into (4a) the condition $(g^A_n Z_o - R^A_n Y_o)\cos(2\beta l) = 2\Re e[j\widetilde{\rho^A_n}]\sin(2\beta l)$ is obtained; (4b) is satisfied if $(g^A_n Z_o - R^A_n Y_o)\sin(2\beta l) = 2\Im m[\widetilde{\rho^A_n}]\cos(2\beta l)$; (4c) and (4d) are solved only if $\Gamma^A_{S_{\text{opt}}} = 0$. This last condition on the amplifier is equivalent to $g^A_n Z_o = R^A_n Y_o$ and $\Im m[\widetilde{\rho^A_n}] = 0$, as it can easily be demonstrated by applying the expressions developed for $Y^{IA}_{S_{\text{opt}}}$ in Appendix A to $Y^A_{S_{\text{opt}}}$ when a real characteristic impedance Z_o is considered. Therefore, (4) is valid $\forall l$ only if $\Gamma^A_{S_{\text{opt}}} = 0$. This result is quite obvious: the amplifier is already signal and noise matched at its input port $\Gamma^A_{\text{in}} = \Gamma^{A^*}_{S_{\text{opt}}} = 0$ and a lossy transmission line will transfer the SSNM condition to its input port while affecting the noise figure only.

V. Conclusion

Noise and signal requirements for a distributed or lumped, active or passive matching network set up a system of nonlinear equations. A reciprocal matching circuit must provide four independent complex terms for its signal matrix. Therefore, a microstrip network comprising only two transmission line elements cannot satisfy the requirements. Nonreciprocal networks can satisfy the system. The matching network must comprise resistive elements in order to have complex elements in its $\boldsymbol{T}^I$ matrix. The active network after the matching stage must satisfy the condition $\Gamma_{\text{in}}^A = (\Gamma_{S_{\text{opt}}}^A)^*$ if the source has to deliver its available power and to assure $F^{IA} = F_{\text{min}}^{IA}$ simultaneously. This letter assumes that the second stage (Fig. 1) is designed before defining (3) on the input-matching circuit. A simultaneous design may lead to different results. The SSNM requirements (3a) and (3b) may be relaxed in order to investigate those applications where an extremely low input return loss is not required.

Appendix A

The system of equations is given as (3). Consider equation (3a), which corresponds to $Y_{S_{\text{opt}}}^{IA} = Y_o$. According to [7], after taking real and imaginary parts, the equation can be rewritten as

$$\frac{G_n^{IA}}{R_n^{IA}} + G_{\text{cor}}^{IA^2} = Y_o^2 \tag{5a}$$

$$B_{\text{cor}}^{IA} = 0 \tag{5b}$$

where $Y_{\text{cor}} = G_{\text{cor}}^{IA} + jB_{\text{cor}}^{IA}$ is the correlation admittance of the cascaded network. Since $Y_{\text{cor}}^{IA} = \rho_n^{IA}\sqrt{g_n^{IA}/R_n^{IA}} = \widetilde{\rho_n^{IA}}/R_n^{IA}$, it is possible to write

$$G_n^{IA} = g_n^{IA} - |Y_{\text{cor}}^{IA}|^2 R_n^{IA} = g_n^{IA} - \frac{|\widetilde{\rho_n^{IA}}|^2}{R_n^{IA}} \tag{6}$$

$$G_{\text{cor}}^{IA} = \Re e[Y_{\text{cor}}^{IA}] = \frac{\Re e[\widetilde{\rho_n^{IA}}]}{R_n^{IA}} \tag{7}$$

$$B_{\text{cor}}^{IA} = \Im m[Y_{\text{cor}}^{IA}] = \frac{\Im m[\widetilde{\rho_n^{IA}}]}{R_n^{IA}}. \tag{8}$$

After substituting (6)–(8), system (5) is equivalent to

$$4g_n^{IA}R_n^{IA} + (\widetilde{\rho_n^{IA}} - \widetilde{\rho_n^{IA*}})^2 = (2R_n^{IA}Y_o)^2 \tag{9a}$$

$$\widetilde{\rho_n^{IA}} - \widetilde{\rho_n^{IA*}} = 0. \tag{9b}$$

After expanding R_n^{IA}, g_n^{IA}, and $\widetilde{\rho_n^{IA}}$ from (1), (4a) and (4b) are obtained from (9a) and (9b).

Now consider (3b). $\Gamma_{\text{in}}^{IA} = 0$ is equivalent to $Z_{\text{in}}^{IA} = Z_o$ or in terms of the $\boldsymbol{T}^I$ matrix elements

$$Z_{\text{in}}^{IA} = \frac{B^I + A^I Z_{\text{in}}^A}{D^I + C^I Z_{\text{in}}^A} = Z_o$$

which gives

$$Z_{\text{in}}^A A^I + B^I - Z_o Z_{\text{in}}^A C^I = Z_o D^I - 0. \tag{10}$$

Using $\Gamma_{\text{in}}^A = \Gamma_{S_{\text{opt}}}^{A^*}$ and $Z_{\text{in}}^A = Z_o(1 + \Gamma_{\text{in}}^A)/(1 - \Gamma_{\text{in}}^A)$, (10) reduces to

$$[1 + (\Gamma_{S_{\text{opt}}}^A)^*]A^I + [1 - (\Gamma_{S_{\text{opt}}}^A)^*]B^I Y_o - [1 + (\Gamma_{S_{\text{opt}}}^A)^*]C^I Z_o - [1 - (\Gamma_{S_{\text{opt}}}^A)^*]D^I = 0.$$

Finally, consider (3c). Since the source reflection coefficient is zero, $\Gamma_{\text{out}}^I = S_{22}^I$, which can be rewritten in terms of the $\boldsymbol{T}^I$ elements [8] as

$$S_{22}^I = \frac{-A^I + Y_o B^I - C^I Z_o + D^I}{A^I + Y_o B^I + C^I Z_o + D^I} = \Gamma_{S_{\text{opt}}}^A.$$

The final equation is therefore

$$[1 + \Gamma_{S_{\text{opt}}}^A]A^I - [1 - \Gamma_{S_{\text{opt}}}^A]B^I Y_o + [1 + \Gamma_{S_{\text{opt}}}^A]C^I Z_o - [1 - \Gamma_{S_{\text{opt}}}^A]D^I = 0.$$

Appendix B

The noise figure can be expressed as

$$F = F_{\text{min}} + \beta\frac{|\Gamma_S - \Gamma_{S_{\text{opt}}}|^2}{1 - |\Gamma_S|^2} \tag{11}$$

where $\beta = R_n/Z_o/|1 + \Gamma_{S_{\text{opt}}}|^2$, while the available gain as

$$G_{\text{av}} = \frac{1 - |\Gamma_S|^2}{|1 - S_{11}\Gamma_S|^2}|S_{21}|^2\frac{1}{1 - |\Gamma_{\text{out}}|^2} \tag{12}$$

both as functions of the scattering parameters of the stage. Here $\Gamma_{\text{out}} = (S_{22} - \Delta\Gamma_S)/(1 - S_{11}\Gamma_S)$ and $\Delta = S_{11}S_{22} - S_{12}S_{21}$. For a passive noisy network $F = 1/G_{\text{av}}$ holds so that (11) and (12) can be equated. The result is

$$F_{\text{min}} = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2 \pm \alpha}{2|S_{21}|^2}$$

$$\Gamma_{S_{\text{opt}}} = \frac{(S_{11} - S_{22}^*\Delta)^*}{\beta|S_{21}|^2}$$

$$\beta = \frac{1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2 \pm \alpha}{2|S_{21}|^2}$$

$$\alpha = \sqrt{(1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2)^2 - 4|S_{11} - S_{22}^*\Delta|^2}.$$

The sign providing $|\Gamma_{S_{\text{opt}}}| < 1$ and $F_{\text{min}} > 1$ should be chosen.

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